SM3 HW2.3: Rational Root Thm

Memorize: Given a polynomial with lead coefficient q and constant p, the possible rational roots are given by $\pm\frac{factors\ of\ p}{factors\ of\ q}$.

Vocab: lead coefficient, constant, rational, synthetic division

For questions 1-3, state the possible rational roots:

1) $a(x) = x^4 + x^2 + 2x - 3$ 2) $b(x) = 2x^2 - 5x + 3$ 3) $c(x) = 4x^6 - x^5 + 3x^3 - 2x + 10$

$\pm 1, \pm 3$ $\pm 1, \pm 3, \pm\frac{1}{2}, \pm\frac{3}{2}$ $\pm 1, \pm 2, \pm 5, \pm 10, \pm\frac{1}{2}, \pm\frac{1}{4}, \pm\frac{5}{2}, \pm\frac{5}{4}$

For problems 4-13, find all of the zeros of each function:

4) $f(x) = x^3 - 3x - 2$ 5) $g(x) = x^3 + x^2 - 80x - 300$ 6) $h(x) = x^3 + 4x^2 + 3x$

$x = \{-1, 2\}$ $x = \{-6, -5, 10\}$ $x = \{-3, -1, 0\}$

7) $j(x) = 2x^3 - 15x^2 + 31x - 12$ 8) $k(x) = 2x^3 - x^2 - 15x + 18$ 9) $l(x) = 6x^3 - 5x^2 - 2x + 1$

$x = \left\{\frac{1}{2}, 3, 4\right\}$ $x = \left\{-3, \frac{3}{2}, 2\right\}$ $x = \left\{-\frac{1}{2}, \frac{1}{3}, 1\right\}$

10) $m(x) = x^4 - 5x^2 - 36$ 11) $n(x) = x^3 - 4x^2 + 6x - 4$ 12) $p(x) = x^3 - 5x^2 + 7x + 13$

$x = \{-3, 3, -2i, 2i\}$ $x = \{2, 1 - i, 1 + i\}$ $x = \{-1, 3 - 2i, 3 + 2i\}$

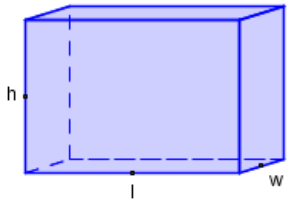
For questions 13-15, find a third degree polynomial with rational coefficients that has the given roots.

13) $x = \{-3, 2, 1\}$ 14) $x = \{2i, -2i, 3\}$ 15) $x = \{0, 5, -6\}$

$x^3 - 7x + 6$ $x^3 - 3x^2 + 4x - 12$ $x^3 + x^2 - 30x$

A failing student is determined to not be particularly useful to society. With a zap of the transmogrification ray, the student is changed into a student-sized goldfish, suited at least for entertainment purposes. The tank used to contain the student-fish has the following properties:

- The height must be 2 ft longer than the width
- The length must be 4 ft longer than the width
- The volume ($V = lwh$) of the tank must be 105 ft^3



16) Build polynomial $v(w)$ by replacing l, h with expressions in terms of w.

$$l = w + 4; h = w + 2$$

$$V(w) = (w + 4)(w)(w + 2) = (w^2 + 4w)(w + 2) = w^3 + 6w^2 + 8w$$

17) Find all roots of $v(w)$.

$w^3 + 6w^2 + 8w = 105$ So, $w^3 + 6w^2 + 8w - 105 = 0$ is what we need to find roots for.

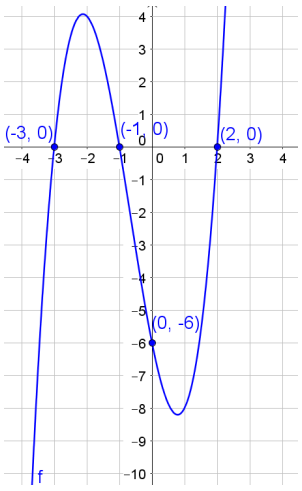
$$w = \left\{3, \frac{-9 \pm i\sqrt{59}}{2}\right\}$$

18) Which root must be the width of the tank? Write a sentence or two that explains why the other roots are not suitable.

We should use the real solution, $w = 3$, because we are not capable of making a tank with imaginary side lengths

19) Determine the potential roots the function may have. Find all of the roots of the function. Then, graph the function. State the domain of the function. Estimate where the function is increasing or decreasing.

$$f(x) = x^3 + 2x^2 - 5x - 6$$



Potential Roots: $x = \{\pm 1, \pm 2, \pm 3, \pm 6\}$

Actual Roots: $x = \{-3\ , -1\ , 2\ \}$

$D_f = \boldsymbol{R}$

Increasing Interval: $(-\infty, -2.1) \cup (0.78, \infty)$
Decreasing Interval: $(-2.1, 0.78)$
(approximately; close answers are acceptable)